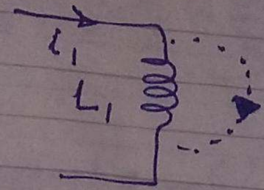
1

ملخص القوانين

1 Self inductance



a) v_{L_1} (induced emf) $= N_1 \frac{d\phi_1}{dt}$

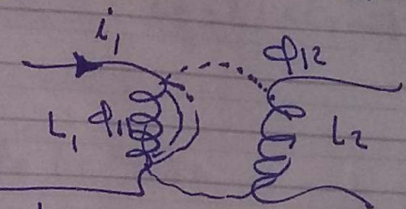
$= L_1 \frac{di_1}{dt}$

b) $L_1 = N_1 \frac{d\phi_1}{di_1}$

→ For linear sy $L_1 = N_1 \phi_1 / i_1$

→ for sinusoidal $v_{L_1} = L_1 \frac{di_1}{dt} = j\omega L_1 i_1$

2 Mutual inductance



a- $M = K\sqrt{L_1 L_2}$ henry

where b- $M = \underbrace{\frac{N_1 d\phi_{21}}{di_2}}_{M_{21}} = \underbrace{\frac{N_2 d\phi_{12}}{di_1}}_{M_{12}}$

$\phi_1 = \phi_{11} + \phi_{12}$ (leakage, linkage)

$\phi_2 = \phi_{22} + \phi_{21}$

c- $K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$ = coupling coeff ≤ 1

dependent
- Space bet^n coils
- μ of medium bet^n coils
- orientation of axis

d- $\cancel{X}_m = k\sqrt{X_{L_1} X_{L_2}}$ Ω

3 self inductance Parameters

$L = \frac{N^2 \mu A}{L}$ (henery)

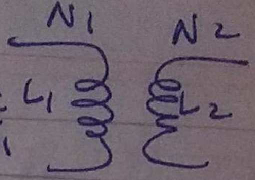
- N → No. of Turns
- A → area
- L → length of coil

$\mu = \mu_0 \mu_r$

- μ_0 → permeability of free space $4\pi \times 10^{-7}$
- μ_r → property of iron ...

3) Transformer

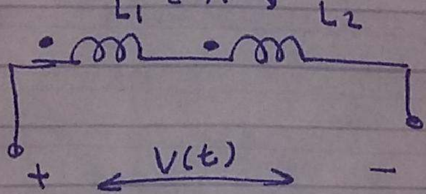
→ Turns Ratio $= \frac{N_1}{N_2} = \frac{N_p}{N_s} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$



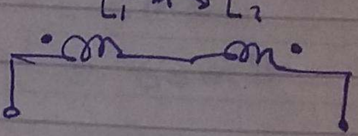
→ $\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}}$

Magnetic coupled coils

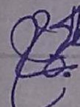
4 sign of M (series/parallel) .. dot convention



$L_{t_1} = L_1 + L_2 + 2M$



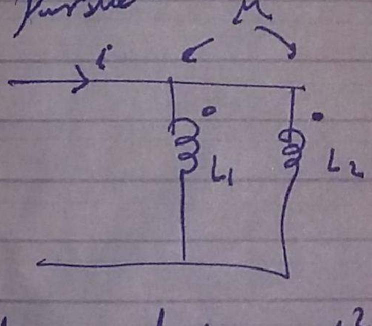
$L_{t_2} = L_1 + L_2 - 2M$

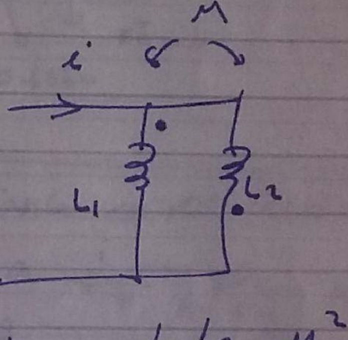


اتجاه دوران

Note $L_{T_1} > L_{T_2}$

Parallel





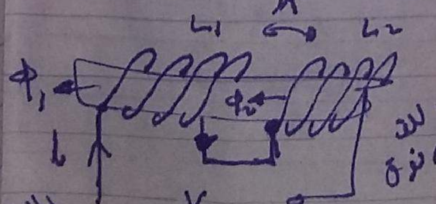
$L_{t_1} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

$L_{T_2} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

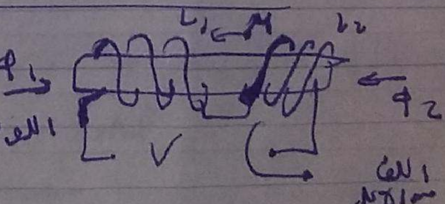
$L_{t_1} > L_{t_2}$

سيريز

شكل آخر لـ series



$L_{eq} = L_1 + L_2 + 2M$



$L_{eq} = L_1 + L_2 - 2M$

(3)

Sheet (6)

1) one coil of magnetically coupled pair has current 5A
Φ_{11}, Φ_{12} are 0.2mwb , 0.4mwb Respectively
If $N_1 = 500$, $N_2 = 1500$, Find L_1, L_2, M, K?

Sol

$\rightarrow \Phi_1 = \Phi_{11} + \Phi_{12} = 0.2 + 0.4 = 0.6 mb$

$\rightarrow L_1 = \frac{N_1 \Phi_1}{I_1} = \frac{500(0.6)}{5} = 60 mH$

$\rightarrow K = \frac{\Phi_{12}}{\Phi_1} = \frac{0.4}{0.6} = 0.6667$

$\rightarrow M = \frac{N_2 \Phi_{12}}{I_1} = \frac{1500(0.4)}{5} = 120 mH$

$\rightarrow K = \frac{M}{\sqrt{L_1 L_2}}$ or $M = K\sqrt{L_1 L_2}$

120mH 0.6667 60mH

$\therefore L_2 = 540 mH$

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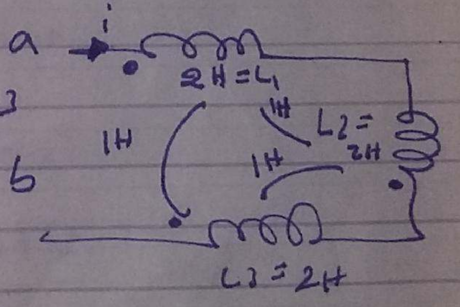
2 find the equivalent inductance across terminals a,b!



$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{13}$
$= 6 - 2 - 2 + 2 = 4H$

OR

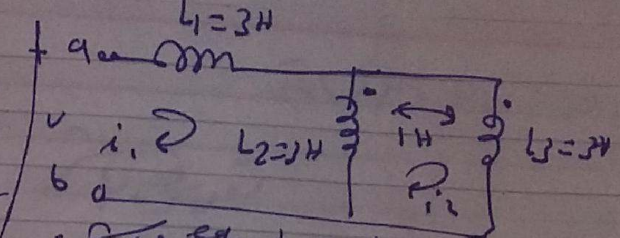
$V = (jwL_1 i - jwM_{12} i - jwM_{13} i) +$
$(jwL_2 i - jwM_{12} i + jwM_{23} i) +$
$(jwL_3 i + jwM_{23} i - jwM_{13} i) = jwi [L_1 + L_2 - 2M_{12} -$
$2M_{13} + 2M_{23}) = jwL_{total}$

4

[13] ط١ بالتيارات

؟ لمَ ؟ ٢

Loop (1)



$= jwL_1 i_1 + (i_1 - i_2) jwL_2 + jwM i_2$

$= i_1 (jwL_1 + jwL_2) + i_2 (jwM - jwL_2) \rightarrow (1)$

ط٢ بالـ eq :

$$L_{23} = \frac{L_2 L_3 - M^2}{L_2 + L_3 - 2M} = 2H$$

$L_{eq} = L_1 + L_{23} = L_1 + L_{23} = 5H$

Loop (2)

$$0 = \left[jwL_3 i_2 - jwM i_2 \right] + (i_2 - i_1) jwL_2 - jwM(i_2 - i_1)$$

العلامة بالسالب

$$0 = i_1 \left[jwM - jwL_2 \right] + i_2 \left[jwL_3 - jwM + jwL_2 - jwM \right] \rightarrow (2)$$

$$\Delta = \begin{bmatrix} jw(L_1 + L_2) & jw(M - L_2) \\ jw(M - L_2) & jw(L_2 + L_3 - 2M) \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} V & jw(M - L_2) \\ 0 & jw(L_2 + L_3 - 2M) \end{bmatrix}, \quad I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_2 = \begin{bmatrix} jw(L_1 + L_2) & V \\ jw(M - L_2) & 0 \end{bmatrix}, \quad I_2 = \Delta_2 / \Delta$$

احسب عوضها اي معامل

note $Z_{in} = Z_{ab} = \frac{\Delta}{\Delta_{11}} \rightarrow$ الصف الاول حذف/ الاول العمود

Δ_i

$\left[\begin{smallmatrix} 4 & 5 \end{smallmatrix} \right]$

$\therefore Z_{in} = \frac{(jw)^2 20}{jw(4)} = jw(5)$

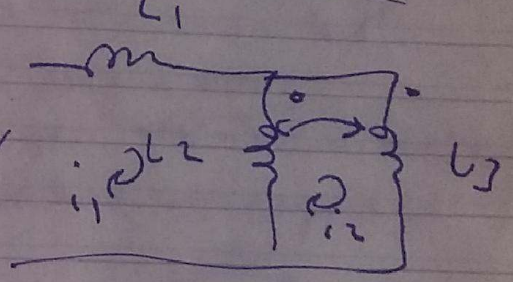
$\therefore L_{total} = 5H$

حل آخر للسابق

تعويض مباشر

$$L_{23} = \frac{L_2 L_3 - M^2}{L_2 + L_3 - 2M} = 2H$$

$$L_{eq} = L_1 + L_{23} = 5H$$



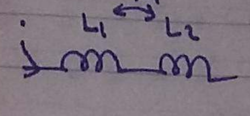
① $V = i_1 (X_{L1} + X_{L2}) - i_2 X_{L2} + i_2 X_M$

② ~~$0 = i_2 (X_{L3} + X_{L2}) + i_1 X_M - 2M i_2 j\omega - i_1 X_{L2}$~~

$$0 = \cancel{i_2 (X_{L2} + X_{L3})} - (i_2 - i_1) j\omega M + (i_2 - i_1) X_{L2} + X_{L3} i_2 - j\omega M i_2 \rightarrow ②$$

6

3 - Calc. mutual inductance of 2 coils of self inductance 100 mH and 200 mH are connected in series to yield a total inductance of 146 mH.



sol/.

$$L_{eq} = L_1 + L_2 \pm 2M$$

$$146m = 100m + 200m \pm 2M$$

$\therefore \pm 2M = (146 - 300)m \Rightarrow \pm M = \pm 77\,mH$

$\therefore M = -77\,mH$ or $\underline{M = 77\,mH}$

$\underline{M\ -ve}$

ملاحظة

$L_1 + L_2 > 146\,mH \therefore$ M-ve

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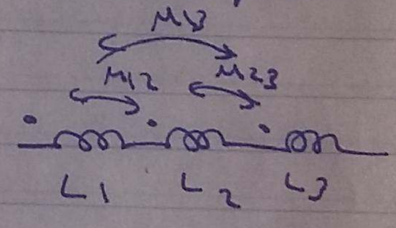
4 3 similar coil wound on along common core, The voltage of mutual inductance between each set of coil is +ve, The self inductance of each coil is 0.2H. The effective inductance of first 2 coils is 0.6H, and of all 3 in series is 1H. (When Terminals of first coil is interchanged, The effective inductance of 3 coils in series be come 0.5H) determine coeff of coupling b/w each set of coil

sol



$\rightarrow L_1 = L_2 = L_3 = L = 0.2H$

$\rightarrow L_{12} = 0.6H$, $L_{total} = 1H$

$\rightarrow L_{12} = \underbrace{L_1}_{0.2} + \underbrace{L_2}_{0.2} + 2M_{12} = 0.6 \quad \therefore \boxed{M_{12} = 0.1H}$

$\rightarrow M_{12} = K_{12}\sqrt{L_1 L_2} = K_{12}\sqrt{0.2 \times 0.2} = K_{12} \times 0.2 \quad \therefore \boxed{K_{12} = 0.5}$

$\rightarrow L_{total} = L_1 + L_2 + L_3 + 2M_{12} + 2M_{23} + 2M_{13}$

$1 = 3 \times 0.2 + 2 \times 0.1 + 2M_{23} + 2M_{13} \quad \therefore \boxed{M_{13} + M_{23} = 0.1}$

$\searrow_1$

When Coil $\underline{1}$ غيرنا الأطراف

$\therefore M_{12}, M_{13} \rightarrow -ve$

$\boxed{L_{T new} = 0.5}$

$0.5 = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{13}$

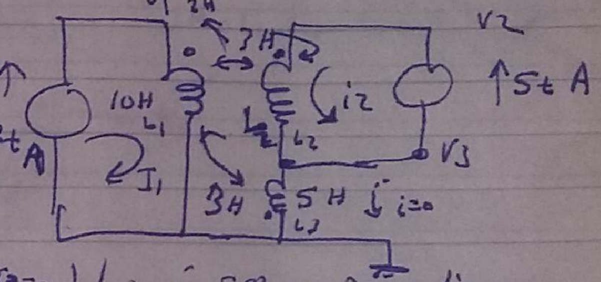
$0.6 = 0.6 - 0.2 + 2M_{23} - 2M_{13}$

$\therefore \boxed{M_{23} - M_{13} = 0.05} \rightarrow 2$

Solve 1, 2 $\therefore M_{13} = 0.025 = K_{13}\sqrt{L_1 L_3} \rightarrow K_{13} = 0.125$

$M_{23} = 0.075 = K_{23}\sqrt{L_2 L_3} \rightarrow K_{23} = 0.375$

$\boxed{5}$ find V_1, V_2, V_3



$\rightarrow V_1 = L_1 \frac{dI_1}{dt} + M_{12}\frac{dI_2}{dt} + \underbrace{M_{13}\frac{dI_3}{dt}}_{0}$

$= 10 \times 2 + 2 \times 5 = 30V$

لو فيه ملف مش جاي منه تيار L_3 $(i_3 = 0)$

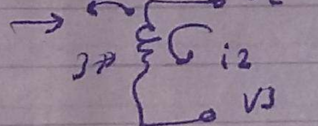
لكن سيكون فيه فولت ناتج

معامل الحث M

بسبب التيار I_1

$\rightarrow V_3 = M_{13}\frac{dI_1}{dt} + \underbrace{L_3\frac{dI_3}{dt}}_{0} = 3 \times 2 = 6V$

$\rightarrow$



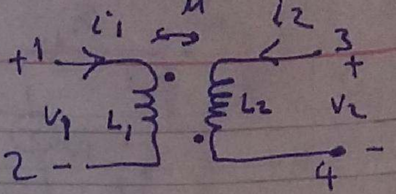
$V_2 - V_3 = L_2\frac{di_2}{dt} + M_{21}\frac{di_1}{dt}$

$V_2 - V_3 = 3 \times 5 + 2 \times 2 = 19V$

$V_2 = V_3 + 19 = 25V$

5 → $L_2 = 2H$, The inductance at 1,2 is 3H when terminals 3, 4 opened

→ And 1H when shorted,
Determine coeff of coupled ckt.



1- $V_{12} = L_1 \frac{di_1}{dt} - \underline{\underline{M}}_{12} \frac{di_2}{dt}$

2- $V_{34} = L_2 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt}$

for 3, 4 $\underline{(o.c)}$ $\longrightarrow i_2 = 0 \quad \therefore di_2 = 0$

$\therefore V_1 = L_1 \frac{di_1}{dt} = jwL_1 i_1 = j X_L \cdot i_1$

$\therefore \boxed{L_1 = 3H}$ $\qquad \therefore V_1$ دالة ثابت $\therefore L_1 = 3H$

for 3, 4 $\underline{S.C}$ $\qquad V_2 = 0$ only

$0 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$

$L_2 \frac{di_2}{dt} = M \frac{di_1}{dt}$

$\therefore \frac{di_2}{dt} = \frac{M}{L_2} \frac{di_1}{dt}$

$\therefore V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = L_1 \frac{di_1}{dt} - \frac{M^2}{L_2} \frac{di_1}{dt} = \left(L_1 - \frac{M^2}{L_2}\right) \frac{di_1}{dt}$

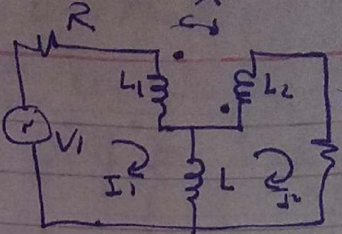
$L_{eq} = \underbrace{L_1}_{3} - \frac{M^2}{L_2} = 1H \qquad \therefore 3 - \frac{M^2}{\underbrace{L_2}_{2}} = 1$

$\therefore M = 2H$

$M = K\sqrt{L_1 L_2} \qquad \therefore K = 0.816$

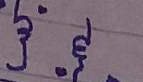
[7] ratio I_1/I_2 = ?! Using { directly coupled ckt / conductively equivalent ckt

sol.



[1] $V_1 = I_1 (R + j\omega(L_1 + L)) - i_2\, j\omega L$ $+ j\omega M\, i_2$

داخل



داخل

$\therefore$ $\boxed{V_1 = I_1 (R + j\omega(L + L_1)) + j\omega(M - L)\, i_2}$

[2] $\boxed{0 = I_2 (R + j\omega(L + L_2)) + j\omega(M - L) I_1}$

$$\Delta = \begin{vmatrix} R + j\omega(L + L_1) & j\omega(M - L) \\ j\omega(M - L) & R + j\omega(L + L_2) \end{vmatrix}$$

$= \boxed{V_1 [R + j\omega(L + L_2)]}$

$$\Delta_1 = \begin{vmatrix} V_1 & j\omega(M - L) \\ 0 & R + j\omega(L + L_2) \end{vmatrix}$$

$= \boxed{V_1 [R + j\omega(L + L_2)]}$

$$\Delta_2 = \begin{vmatrix} R + j\omega(L + L_1) & V_1 \\ j\omega(M - L) & 0 \end{vmatrix}$$

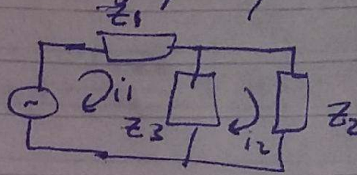
$= -V_1 (j\omega(M - L)) = \boxed{V_1 (j\omega(L - M))}$

$I_1 = \Delta_1/\Delta$ $\quad$ $I_2 = \Delta_2/\Delta$

$I_1/I_2 = \Delta_1/\Delta_2 = \boxed{\dfrac{R + j\omega(L + L_2)}{j\omega(L - M)}}$

ثانياً: دائرة

Conductively coupled equivalent



$V = (Z_1 + Z_3)\, i_1 - Z_3\, i_2 \longrightarrow (3)$

$0 = -Z_3\, i_1 + (Z_2 + Z_3)\, i_2 \longrightarrow (4)$

compare 3, 4 by 1, 2

$\therefore Z_1 + Z_3 = R + j\omega(L_1 + L) \longrightarrow (5)$

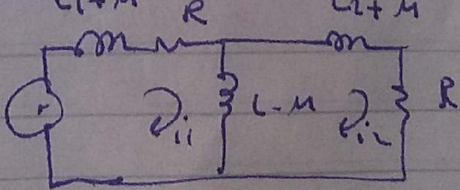
$-Z_3 = j\omega(M - L)$

or $\boxed{Z_3 = j\omega(L - M)} \to (6)$

$\therefore \boxed{Z_1 = R + j\omega(L_1 + M)} \to (7)$

$Z_2 = R + j\omega(L_2 + L) - j\omega(L - M)$

$\therefore \boxed{Z_2 = R + j\omega(L_2 + M)} \to 8$



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